A unified model for the optimal management of electrical and thermal equipment of a prosumer in a DR environment

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Due to the recent trend toward the decentralization of energy production, i.e., the presence of small renewable energy plants directly connected to the grid, end users are fostered to proactively participate to the energy management by joining the so-called demand response programs. In addition, groups of end users are encouraged to form energy districts in which the energy is exchanged locally and negotiated, without intermediaries, directly with the energy providers. In this paper, a demand response program is envisioned at the energy district level where an aggregator dynamically determines the energy prices on the basis of market conditions, and a Cloud component is in charge of supplying high-level aggregated information. End user dwellings are purposely equipped in order to retrieve information from the Cloud so as to establish the best energy management strategy. This strategy is determined by solving an optimization problem called "prosumer problem". The model underlying the prosumer problem takes into account both thermal and electrical components at the same time. Experimental results show the benefits of this unified thermal and electrical model in terms of improved energy efficiency and reduction of energy costs.

Index Terms—**Demand Response, Prosumer, Energy District, Smart Grid, Home Automation System**

I. INTRODUCTION

The strong decentralization of energy production, especially from non-programmable renewable sources (nPRS), obtained with the utilization and interconnection of small plants, has placed the end user at the centre of the energy system management. The end user has taken the role of a "prosumer", being at the same time producer and consumer of thermal and electrical energy. While this new bivalent role has clear advantages (on-site production, lower transport losses, reduced dependence on fossil fuels, etc.) [1], the distributed generation from nPRS causes additional injections of energy into the grid, which can bring to stability and safety problems for the operations of the grid itself [2][3]. As a consequence, the end-user needs to be involved in the management of the grid, and appropriate strategies should be adopted in order to maintain the balance between generation and consumption of energy, and avoid spikes of energy demand or excessive injections of energy produced but not consumed. To cope with these issues, the end-user is often encouraged to participate in *demand response* (DR) programs [4].

A. State-of-the-art

The existing literature on DR programs is wide and heterogeneous. According to the definition of DR program in [5], the end-users that join a DR program, modify their electricity consumption pattern in terms of timing, level of instantaneous demand, or total electricity consumption [6][7], in order to reduce their peak demand, save money and have a more eco-friendly standard of life [8][9][10]. Due to the possible simultaneous presence of electric and thermal devices – representing electric and thermal loads – and nPRS generators – e.g., photovoltaic (PV) plants, heat pumps (HP) and combined generators (CHP) – the problems of electric and thermal management are strictly interconnected. Some recent papers cover the topic of thermal and electrical management at the user level. The work presented in [11] aims to minimize the electric and thermal cost over one day, assessing the level of thermal comfort as the objective function of a linear optimization problem. The results of this article were evaluated considering a user equipped with HPs, a micro CHP, electric loads and thermal loads depending on the external temperature. The thermal inertia of the dwelling walls was considered as a thermal storage system. In [12], the goal is to keep the cost of the electricity bill below a threshold chosen by the user, considering that a lower threshold corresponds to a higher discomfort level. Starting from a diversified set of loads (flexible, non-flexible, thermal, curtailable and critical appliances), the optimization is performed through successive stages, in which the number of interruptible loads gradually increases and, as a consequence, the level of user dissatisfaction grows. The work presented in [13] considers the presence of electrical and thermal loads and of electric vehicles used as storage systems. For each load, a flat profile is supposed, while production profiles of renewable sources are estimated using appropriate forecast models. All this data is passed to a two-stage optimization model that maximizes the algebraic sum of the revenues obtained with energy production and of the costs of energy purchase. The work in [14] analyzes the management of electric and thermal loads (controllable/uncontrollable and deferrable) and the management of production from controllable sources through two approaches: the first schedules the controllable loads, considering the user preferences and the energy prices; the second reallocates the loads depending on the availability

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of energy surplus from renewable sources. The management is done by evaluating the end-user comfort in terms of the preferred temperatures setting and/or the start of the loads operation. In [14], the end-users manage their equipment by using local controllers connected to a single central controller, thus forming a "cooperative neighbourhood". In [15], a local energy box independently manages the usage, the storage and the sale of energy in response to the real-time conditions of the grid, allowing the user to balance the level of comfort and minimize the electric bill. The work presented in [16] analyzes a management model of electric and thermal loads, based on the observance of some consumption limits. The control takes place in real-time and requires the active participation of the end-users, who must respond to the outcome of the optimization, by accepting or rejecting the proposed schedule. The authors of [17] propose a demand response program for prosumers based on a management model of consumption and generation, and solve the so-called *prosumer problem*. According to the classification of various types of DR programs in [7], [8] and [18], the management model in [17] is a "price-based" and "time-of-use" program that minimizes the cost of energy consumption. Prosumers are arranged in energy districts managed by a centralized coordinator, referred to as *aggregator*, which is in charge of the activities regarding the energy purchase and sale within the district and between the district and the energy market. The aggregator has the task of applying suitable strategies for improving the energy efficiency thus reducing the costs for all the users of the district.

B. Novelty

Starting from the same context as the one described in [17], this paper presents a DR program envisioned at the energy district level where the aggregator dynamically determines the energy prices on the basis of energy market conditions. The whole district is supported by a Cloud component, which is in charge of supplying high-level aggregated information to the prosumers so as to allow them to participate in the DR program and choose the best energy strategy. Each prosumer that belongs to this district and participates in the DR program, is equipped with a Smart Energy Box, as described in [19], which manages the prosumer's devices, retrieves the aggregated information from the Cloud component, and elaborates and actuates a proper strategy for minimizing the energy cost.

This work introduces the *unified prosumer problem*, an enhanced version of the prosumer problem described in [17]. The new version starts from a unified model for optimizing, at the same time, the electrical and thermal energy management of a prosumer. The objective is to optimally schedule loads, generation and storage of a set of electric and thermal devices, in order to minimize the cost associated with the supply of electricity and heat, while taking into account a set of user-defined energy preferences. The main enhancements introduced in this work with respect to [17] are summarized in the following. First of all, the thermal component in [17] only consists of a microCHP whereas this work also considers the gas boiler, the heat pump and the solar panel. In addition, our unified prosumer problem also models the possibility to sell energy to the grid so as to maximize the revenues besides minimizing the costs. Another important enhancement refers to the modeling of the electric and thermal storages, not present in [17]. Moreover the thermal load is determined using the algorithm reported in [20] that allows a good compromise between the computational burden and the accuracy of results in simulating the dynamic behavior of the buildings, improving the thermal load model presented in [17]. Finally, while the optimization in [17] only concerns the electric loads, the unified prosumer problem introduced here also produces the optimal scheduling for the controllable plants and the electric and thermal storage systems.

This unified model exhibits better performance, in terms of energy cost reduction, with respect to the approaches in which the electrical and thermal aspects are managed separately. Differently from [12][14][15][16], in which the management models try to achieve a trade-off between energy cost reduction and the user energy comfort, our approach allows to totally satisfy the user preferences. In addition, differently from [16], the proposed model allows to manage the prosumer equipment almost autonomously with a minimal involvement of end-users, who are only in charge of setting their load schedule preferences. The model takes into account various types of loads (both schedulable and non-schedulable) and different types of power generation systems (using traditional and renewable sources, programmable and non-programmable). Moreover, other additional devices are considered, i.e., the heat pump, the gas boiler and the solar panel system. Finally, both electrical and thermal storage systems are considered.

C. Structure of the paper

The paper is organized as follows. Section II describes the characteristics of a thermal/electrical prosumer and its interaction with the energy district. We detail the equipment of a prosumer by illustrating the main parameters and the basic characteristic equations. Then we detail the daily costs of prosumers, specify the information that is retrieved from the Cloud component, and describe the information that needs to be supplied by the dwellings inhabitants. Starting from the parameters and variables introduced in Section II, we formalize the *unified prosumer problem* model in Section III and discuss experimental results in Section IV. Section V concludes the paper.

II. THERMAL/ELECTRICAL PROSUMER IN AN ENERGY DISTRICT

Before describing the characteristics of thermal/electrical prosumers, we introduce the architecture of the energy district, which is outlined in Figure 1. The involved coarse-grained entities are: a *prosumer*, a centralized *aggregator* and a Cloud component called *Cloud service provider*.

Each prosumer hosts the following equipment:

- a **nanogrid system**, which manages the energy exchange between the prosumer and the *distribution grid*, the local energy *production plants* and the *storage systems*;
- a **home automation system**, which manages the activation and deactivation of the electrical *loads* of the dwellings and the thermal devices;

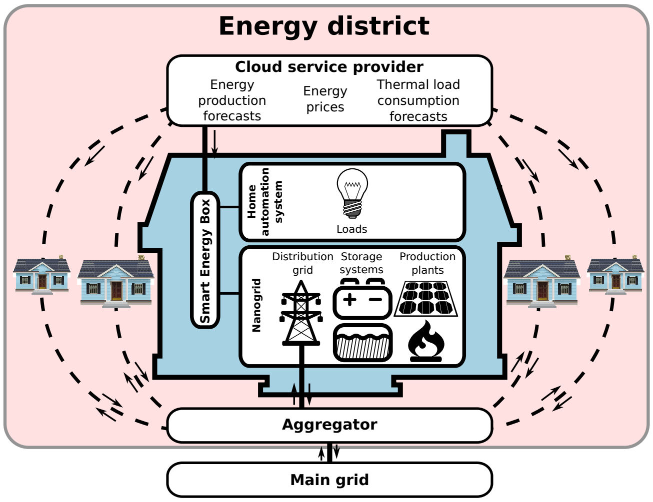

Fig. 1. Architecture of the energy district

- a **smart energy box**, which enables the interaction with the Cloud service provider and supervises both the nanogrid and the home automation system. It runs the so-called unified prosumer problem discussed in the following.

The aggregator is in charge of managing the energy exchanges among the prosumers and between the whole district and the *distribution grid*. The Cloud service provider supplies information to the prosumers by executing a set of algorithms requiring computational resources that are beyond the capabilities of the smart energy box. In particular, these algorithms provide, on a daily basis, information concerning:

- the hour-by-hour *energy production forecasts* based on the weather forecast and the physical characteristics of the generation plants [21];
- the hour-by-hour *energy prices* determined by the conditions of the energy market;
- the hour-by-hour *thermal load consumption forecasts* elaborated in order to guarantee some predefined thermal conditions.

This information is computed by the Cloud service provider every day for the following day and is sent at a predetermined hour to all the prosumers. The smart energy boxes of the prosumers use this information to elaborate a proper working plan for the nanogrid and the home automation system, which optimizes the energy consumption and reduces the costs. In particular, each smart energy box solves the so called *unified prosumer problem* taking as input: the energy production forecasts, the energy prices and the thermal load consumption forecasts. The output of the *unified prosumer problem* is the optimal scheduling for: (i) the activation/deactivation of the electrical loads, (ii) the charging/discharging of the storage systems, (iii) the amount of energy imported/exported from/to the grid, and (iv) the operations of controllable plants (e.g. the HP and the micro CHP). Since the purpose of the scheduling is to respond to the needs of the dwellings inhabitants, the *unified prosumer problem* takes into account a set of user preferences that are supplied by means of a graphical user interface (GUI) exposed by the smart energy box.

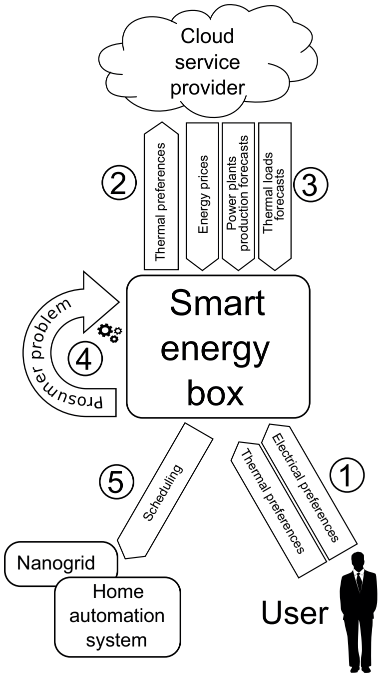

Fig. 2. Daily information flow

In Figure 2, the daily activities performed by a smart energy box are summarized. Four entities are involved: the smart energy box, the user, the cloud service provider and the prosumer devices, managed by the nanogrid and the home automation system. Every day, before a predetermined hour, users set their thermal and electric preferences for the following day (label 1 in Figure 2). The smart energy box sends the thermal preferences, i.e., the desired set points of temperature, to the Cloud service provider (label 2), which uses the preferences to compute the thermal load forecast. Moreover, the Cloud service provider computes the production forecast of non-controllable power plants and determines the energy prices for importing/exporting electric energy. All this information is then sent to the smart energy box (label 3), which runs the *unified prosumer problem* (label 4) and stores the results in a local database. On the following day, the smart energy box uses these results and actuates the schedules on the nanogrid and the home automation system (label 5). In particular, the actuation involves the electric loads and the controllable plants, e.g., the gas boiler, the microCHP and the heat pump. During the following day, there could be some mismatch between forecasts and real value of produced/absorbed energy; such mismatch may cause imbalances between the forecasted profile exchanged with the grid and the real profile. How to reduce such mismatch is out of topic of the paper, however, this issue can be tackled by a suitable real-time control such as the one reported in [22]. Information about the IT architecture of the energy district, and the adopted hw/sw solutions, is available at [23].

A. Loads, storage systems and energy source plants of a prosumer

In this section, we summarize the equipment of a prosumer. The equipment can be categorized into four main groups: (i) electrical and thermal loads (e.g. lightning, appliances, heating systems and so on), (ii) electrical and thermal storage systems, (iii) renewable energy sources such as PV plants, solar thermal panels and biomass-based micro CHP generators and (iv) traditional energy sources such as the gas boilers, the heat pumps, and the points of delivery of the distribution grid.

In the proposed model we use an hour as temporal granularity, i.e., the power consumed by the loads as well as the power produced by the plants or exchanged with the storage systems is assumed to be constant within each hour h of the day, with $h \in H$, where H is the set of hours of a day. In the following, the equipment is described in detail by introducing the main parameters and the characteristic equations.

1) Loads

The loads of a generic prosumer can be divided into: L, the set of electrical loads and T, the set of thermal loads. For the sake of simplicity, we assume that each electrical load $a \in L$ can be either turned on or turned off. A load a is turned on for a given number of hours, θ_a, during which it operates at its rated power P_a^{rat}, while it is turned off during the rest of the day. Let us define E_a as the hourly energy required by a load a. It can be computed as $E_a = P_a^{rat} * \Delta t$ with $\Delta t = 1h$.

The electrical loads are further divided into a set A of schedulable loads and a set B of non-schedulable loads. A schedulable load is a load that can be activated at any time within a user-defined time interval (see Section II-D), whereas a non-schedulable load must be activated at a fixed hour of the day. Washing machines and dish washings are examples of schedulable loads, while refrigerators are examples of non-schedulable loads. Each thermal load corresponds to a room that needs to be separately conditioned in a multi-zone heating system. The thermal energy profile is computed by the Cloud service provider on the basis of the user-defined thermal preferences (temperature set-points), the weather forecast and the physical and geometrical characteristics of the envelope. Thermal loads are computed exploiting the dynamic model 5R1C described in the standard EN ISO 13790 [20].

The hourly energy profile of an electrical load $a \in L$ and of a thermal load $t \in T$ at a given hour h are respectively referred to as x_a^h and x_t^h.

2) Electric storage system

The electric storage system is used to store the surplus of energy and exploit it when an energy deficit happens. During the charging/discharging phases, there is an energy loss which is modeled by the charging/discharging efficiency factors, respectively η_{cha} and η_{dis}. Other characteristic parameters are: the maximum/minimum percentages of the state of charge, SOC_{max} and SOC_{min}, the maximum capacity of the storage, C_{max}, and the maximum hourly charging/discharging amounts of energy, E_{cha}^{max} and E_{dis}^{max}. Let us define E_{dis}^h and E_{cha}^h as, respectively, the drawn and the stored energy during the hour h. We also define $E_{STO_{el}}^*$ as the residual energy of the day before.

The constraints related to the maximum drawn and stored hourly energy can be expressed by the inequalities (1) and (2).

$$0 \leq E_{cha}^h \leq E_{cha}^{max} \quad \forall h \in H \tag{1}$$

$$0 \leq E_{dis}^h \leq E_{dis}^{max} \quad \forall h \in H \tag{2}$$

The inequalities (3) and (4) express the fact that the total stored energy at each hour $h \in H$ is within the minimum and the maximum state of charge.

$$E_{STO_{el}}^* + \sum_{i=0}^{h} E_{cha}^i - \sum_{i=0}^{h} E_{dis}^i \geq SOC_{min} \cdot C_{max} \quad \forall h \in H \tag{3}$$

$$E_{STO_{el}}^* + \sum_{i=0}^{h} E_{cha}^i - \sum_{i=0}^{h} E_{dis}^i \leq SOC_{max} \cdot C_{max} \quad \forall h \in H \tag{4}$$

3) Thermal storage system

The thermal storage system is typically an insulated water tank which exploits the water thermal capacity in order to temporary store the surplus of thermal energy. The main parameters of the storage system are V, the volume of the tank, and the thermodynamic properties of the heat transfer fluid, i.e., the fluid density ρ and the specific heat C_p. Other parameters of the thermal storage system are: $T_{STO_t}^{max}$ and $T_{STO_t}^{min}$, which are respectively the maximum and the minimum admissible temperature, and $\Delta T_{STO_t}^{max}$, the maximum temperature variation in an hour. Let us define $E_{STO_t}^h$ as the energy exchanged by the storage system during the hour h, with the convention that $E_{STO_t}^h$ is positive when the storage is discharged and negative when it is charged. We also define $E_{STO_t}^*$ as the thermal energy remaining in the thermal storage system at the end of the previous day. The inequality (5) ensures that the temperature variation does not violate the maximum allowed value, whereas the inequality (6) forces the temperature to be inside its minimum-maximum range.

$$-\rho V C_p \cdot \Delta T_{STO_t}^{max} \leq E_{STO_t}^h \leq \rho V C_p \cdot \Delta T_{STO_t}^{max} \quad \forall h \in H \tag{5}$$

$$0 \leq \sum_{i=0}^{h} E_{STO_t}^i + E_{STO_t}^* \leq \rho V C_p \cdot (T_{STO_t}^{max} - T_{STO_t}^{min}) \quad \forall h \in H \tag{6}$$

4) PV Plant

The PV plant is an nPRS electric generator. The electrical energy produced during the hour h is referred to as E_{PV}^h.

5) Solar thermal panels

It is an nPRS thermal generator typically consists in a panel that transfers primary solar energy to a thermal fluid (a water-glycol mixture) forced to flow in a closed circuit. The thermal fluid transfers the captured energy to an insulated water tank by means of a heat exchanger. We define the thermal energy produced at hour h as $E_{t_{SOL}}^h$.

6) Micro-CHP generator

A micro-CHP generator is a system able to produce thermal and electrical energy at the same time. This kind of system is based on different primary sources such as fossil fuels, natural gas, organic material, wood, crop residues, which are usually called biomasses. In this work we focus on an innovative micro-CHP generator consisting of a biomass boiler on which a free piston Stirling engine and a linear generator are mounted. This micro-CHP generator is characterized by the electrical rated power P_{CHP}^{rat} and the thermal rated power $P_{CHP_t}^{rat}$, which correspond, respectively, to the maximum electrical and thermal powers that can be generated. The ratio between electrical and thermal power is called co-generation factor, F_{CHP}, and it can be computed using the rated powers, i.e., $F_{CHP} = (P_{CHP_t}^{rat})/(P_{CHP}^{rat})$. The hourly maximum electrical energy, E_{CHP}, can be easily derived from the rated power: $E_{CHP} = P_{CHP}^{rat} \cdot \Delta t$, with Δt=1h. Other parameters of the combined generator are the thermal (electrical) efficiency, η_{CHP}^{el} (η_{CHP}^{t}), obtained as the ratio between the produced electrical (thermal) power and the power supplied by biomass combustion. The latter depends on the *lower heating value* of the biomass, $LHV_{biomass}$, i.e., the amount of thermal energy produced per biomass unity (kg). Finally, E_{CHP}^{h} is defined as the hourly electrical produced energy at hour h. The latter amount of energy must be lower than the hourly maximum electrical energy E_{CHP}, as expressed by the inequality (7).

$$0 \leq E_{CHP}^{h} \leq E_{CHP} \quad \forall h \in H \tag{7}$$

The thermal energy produced at a given hour can be derived by multiplying the co-generation factor by the produced electrical energy: $F_{CHP} \cdot E_{CHP}^{h}$.

7) Connection to the distribution grid

The prosumer is connected to the grid by means of the so-called Point Of Delivery (POD) through which it exchanges electrical energy with the other prosumers of the energy district and with the distribution grid (via the aggregator). The rated power P_{rated}, established with the local retailer, defines the maximum power that can be supplied by the distribution grid. We define E_{imp}^{h} and E_{exp}^{h} as the imported and the exported energy at hour h, respectively. The maximum amount of energy that can be imported in an hour can be computed as $E_{grid}^{max} = P_{rated} \cdot \Delta t$ with Δt=1h, so the imported energy at a given hour is constrained by the inequality (8):

$$E_{imp}^{h} \leq E_{grid}^{max} \quad \forall h \in H \tag{8}$$

The exported energy is inherently limited by the rated powers of the plants that are typically sized not to overflow the maximum amount of exportable power, established with the local retailer.

8) Gas boiler

A typical gas boiler produces thermal energy by natural gas combustion. It is characterized by the thermal rated power P_{NG}^{rat}, which corresponds to the maximum thermal power that can be generated. The hourly maximum thermal energy, E_{NG}, can be derived from the rated power: $E_{NG} = P_{NG}^{rat} \cdot \Delta t$ with Δt=1h. Another important parameter is the thermal efficiency, η_{NG} obtained as the ratio between the produced thermal power and the power supplied by natural gas combustion. The latter depends on the lower heating value of the gas, LHV_{gas}, i.e., the amount of thermal energy produced per gas unity (Nm^3). Finally, E_{NG}^{h} is defined as the energy produced at hour h. The latter energy must be lower than the hourly maximum energy E_{NG}, as expressed by the inequality (9):

$$0 \leq E_{NG}^{h} \leq E_{NG} \quad \forall h \in H \tag{9}$$

9) Heat pump (HP)

A HP transfers thermal energy from the outside to the inside and vice versa, consuming electrical energy. The ratio between the thermal rated power $P_{HP_t}^{rat}$, i.e., the maximum power that can be produced, and the electrical rated power P_{HP}^{rat}, i.e., the maximum power that can be consumed, is called Coefficient Of Performance, COP. We define E_{HP}^{h} as the electrical energy consumed at hour h. The maximum amount of hourly energy that can be consumed is derived from the electrical rated power: $E_{HP} = P_{HP}^{rat} \cdot \Delta t$ with Δt=1h, so the consumed electrical energy is constrained as specified in inequality (10):

$$E_{HP}^{h} \leq E_{HP} \quad \forall h \in H \tag{10}$$

The thermal energy produced at hour h can be computed as $COP \cdot E_{HP}^{h}$.

B. Daily energy cost for the prosumer

The daily energy cost incurred by a prosumer is related to the quantity of primary energy sources utilized, i.e., the biomass of the micro-CHP generator, the natural gas used in the gas boiler and the energy imported from the distribution grid. The cost of a kilogram of biomass is denoted as $c_{kg_{biomass}}$. The cost of a thermal kWh produced by the micro-CHP generator can be computed as:

$$C_{kWht\_CHP} = c_{kg_{biomass}} \cdot \frac{1}{F_{CHP} \cdot \eta_{CHP}^{el} \cdot LHV_{biomass}}$$

An analogous computation applies to the gas boiler: the cost of a Nm^3 of natural gas is denoted as $c_{Nm^3_{gas}}$ and the cost of a thermal kWh produced by the gas boiler is:

$$C_{kWht\_NG} = c_{Nm^3_{gas}} \cdot \frac{1}{\eta_{NG} \cdot LHV_{gas}}$$

The cost of an electrical kWh imported from the grid varies with the hour h and is denoted as c^h. The prosumer can also export electrical energy to the grid. In this case, the energy is sold at a given hourly price p^h. For the sake of simplicity, we do not consider any installation or maintenance cost related to the plants and, as a consequence, there are no costs for the energy produced by the PV plant and by the solar panels.

C. Day-ahead Cloud information

As mentioned before, each day, at a predetermined hour, the prosumer retrieves from the Cloud service provider the information used to run the *unified prosumer problem*. In particular, the *unified prosumer problem* needs one day-ahead information about the energy costs, the production forecast of the renewable plants and the values of thermal load that fulfil the thermal preferences of the users. In particular, the cost, hour-by-hour, of a kWh imported/exported from/to the grid, c^h/p^h, is computed taking into account the trend of the electrical market prices. The day-ahead production forecasts are elaborated taking into account the weather forecast. In particular, the production forecasts concern the hour-by-hour energy production of the PV and the solar panels, defined respectively as E^h_{PV} and $E^h_{t_{SOL}}$. The values of the thermal loads are also computed by the Cloud service provider taking into account the user-defined temperature set points, the weather forecast and the thermodynamic characteristics of the dwellings.

D. *User preferences*

The solution of the *unified prosumer problem* aims to optimize the operation of the prosumer equipment by taking into account the user preferences about the scheduling of the loads. As described in Section II-A, the set L of electrical loads is divided into two subsets: the set of schedulable loads A and the set of non-schedulable loads B. These sets change dynamically and need to be defined by the user in order to plan which appliances will be activated in the next day, and at which hours. The user can also set the working time of non-schedulable loads by providing the daily energy profile in terms of hour-by-hour energy consumption x^h_a for each load $a \in B$. In practice, x^h_a is set to be 0 for each hour $h \in H$ except for the hours in which the appliance is scheduled to be switched on (in these hours the energy consumption is equal to the hourly energy required E_a). For schedulable loads, the user must define, for each $a \in A$, the duration in hours of the working time θ_a and the range of hours $[\alpha_a, \beta_a]$ to which the working time must belong. In addition, the user can define the temperature set points that are used by the Cloud service provider to compute the thermal loads.

These preferences and user data need to be managed both with an administrative and a technical approach to ensure privacy. User data, specifically the thermodynamic characteristics of each dwelling, is protected by an ad-hoc agreement signed by the user and the district administrator. This agreement specifies that such information cannot be shared with the other users or with external entities, and can only be used by the Cloud service provider for the technical purposes of the district, e.g., obtain the thermal load forecast, which is needed to solve the prosumer problem. From a technical point of view, it is necessary to protect the private data in two ways: (i) the transmission of the data to the district database must be secure; (ii) access to data must be denied to non-authorized users and services. A specialist, possibly in presence of the dwelling inhabitant, fills up a document with all the needed thermodynamic characteristics regarding the dwelling. Then, the specialist uploads the data on the district database. The data upload procedure is done through the database client using the access privileges assigned to the specialist. Once uploaded and stored in the database, the data - unless it is successively updated, using a similar procedure - will only be used by the load forecast service, while it will not be accessed by any other user or service.

III. THE UNIFIED PROSUMER PROBLEM

Starting from the parameters and equations/inequalities introduced in the previous section, we build a linear integer optimization problem consisting of an objective function and a set of inequality/equality constraints. The formalization of the *unified prosumer problem* as a linear integer optimization problem allow us to find the optimal solution by adopting the well-known Branch and Bound algorithm. The optimal solution consists in determining the values of a set of variables that minimize the costs and maximize the revenues, while respecting a set of constraints. In the following, we first identify the variables of the optimization problem, then we detail the constraints and the objective function. The variables are:

1) the hour-by-hour amounts of energy introduced in the previous section, which are related to the controllable plants (i.e., the distribution grid, the electrical and thermal storage systems, the micro-CHP generator, the gas boiler and the HP);
2) a set of auxiliary variables used to model the activation/deactivation of the loads;
3) a set of variables used to model the hourly surplus of thermal energy, as will be clarified in the following.

The first set comprises the following variables, defined $\forall h \in H$ of the next day:

- E^h_{dis}, the energy drawn from the electrical storage;
- E^h_{cha}, the accumulated energy of the electrical storage;
- $E^h_{STO_t}$, the energy exchanged by the thermal storage system;
- E^h_{CHP}, the electrical energy produced by the combined generator;
- E^h_{imp}, the electrical energy imported from the grid;
- E^h_{exp}, the electrical energy exported to the grid;
- E^h_{NG}, the thermal energy produced by the gas boiler;
- E^h_{HP}, the electrical energy consumed by the heat pump.

The auxiliary variables used to model the activation/deactivation of the loads, defined $\forall h \in H$ of the next day and $\forall a \in A$, are the following:

- y^h_a has value 0 if the load a is scheduled to be inactive at the hour h, and has value 1 if the load a is scheduled to be active at the hour h;
- z^h_a has value 1 at the hour h in which the load a is scheduled to be switched on, passing from the inactive state to the active state, and has value 0 at all the other hours;

Starting from these variables, we can model the activation/deactivation of the loads taking into account the user preferences detailed in Section II-D. The equations (11) and (12) force the activation of a load a to occur at a single hour $h \in [\alpha_a, \beta_a - \theta_a + 1]$. The upper bound of the latter interval is set to $\beta_a - \theta_a + 1$, instead of β_a, to ensure that the working time of the load a ends before β_a.

$$\sum_{h=\alpha_a}^{\beta_a-\theta_a+1} z^h_a = 1 \quad \forall a \in A \tag{11}$$

$$z^h_a = 0 \quad \forall h \in H \setminus [\alpha_a, \beta_a - \theta_a + 1] \quad \forall a \in A \tag{12}$$

The equation (13) ensures that the load a is activated exactly for θ_a hours inside the $[\alpha_a, \beta_a]$ preference interval. In the case of the y_a^h variables, differently from the z_a^h variables, there is no need to set the variables to 0 outside the user preference interval. Indeed, the optimization process excludes this possibility because it would lead to a sub-optimal solution: the activation of the load outside the user preference interval would increase the costs.

$$\sum_{h=\alpha_a}^{\beta_a} y_a^h = \theta_a \quad \forall a \in A \tag{13}$$

The set of inequalities (14) forces the load a to operate during its working time without interruptions. Indeed, for all h where z_a^h is 1, the $y_a^h, y_a^{h+1}, ..., y_a^{h+\theta_a-1}$ variables can assume values greater or equal to 1. On the other hand, the equation (13) forces the same variables to assume values lesser or equal to 1. As a consequence, the y_a^h variables are equal to 1 for each hour $h \in [h_a^*, h_a^*+\theta_a-1]$ where h_a^* is the activation hour of the load a, i.e. the hour in which $z_a^h = 1$. Similarly, the equation (13), combined with the inequality (14), also ensures that the y_a^h variables are equal to 0 outside the $[h_a^*, h_a^*+\theta_a-1]$ interval.

$$y_a^h \geq z_a^h, y_a^{h+1} \geq z_a^h, ..., y_a^{h+\theta_a-1} \geq z_a^h \qquad \forall h \in [\alpha_a, \beta_a], \forall a \in A \tag{14}$$

All the introduced energy variables must satisfy the electrical and thermal energy balancing. In particular, equation (15) models the electrical energy balancing, whereas equation (16) models the thermal energy balancing.

$$E_{imp}^h + \eta_{dis} \cdot E_{dis}^h + E_{CHP}^h - E_{exp}^h - \frac{1}{\eta_{cha}} \cdot E_{cha}^h - E_{HP}^h - \sum_{a \in A} y_a^h \cdot E_a = \sum_{b \in B} x_b^h - E_{PV}^h \quad \forall h \in H \tag{15}$$

$$E_{HP}^h \cdot COP + E_{CHP}^h \cdot F_{CHP} + E_{NG}^h + E_{STO_t}^h - X_{diss}^h = \sum_{t \in T} x_t^h - E_{t_{SOL}}^h \quad \forall h \in H \tag{16}$$

In equation (16) we have introduced the third set of variables, used to model the hourly surplus of thermal energy, i.e., the variables X_{diss}^h. More specifically, they are used to model the amount of produced thermal energy that exceeds the thermal loads, which is consumed by a purposely adopted dissipation system. There is no need to introduce similar variables for the electrical part because the electrical energy that exceeds the prosumer requirements can always be injected into the grid. The other constraints of the model are the inequalities introduced in the previous sections that are referred to the equipment, i.e., the inequalities (1)–(10).

Finally, we compute the daily energy cost for the prosumer that is used as the objective function to minimize, see formula (17). The function is expressed as the sum of three positive terms – the daily costs related to the microCHP, the gas boiler and the electrical energy imported from the grid – and one negative term, corresponding to the revenues obtained by selling energy to the grid.

$$min \sum_{h \in H} (C_{kWht\_CHP} \cdot F_{CHP} \cdot E_{CHP}^h + C_{kWht\_NG} \cdot E_{NG}^h + c^h \cdot E_{imp}^h - p^h \cdot E_{exp}^h) \tag{17}$$

The total number of variables is $11 \cdot |H| = 264$ where $|H| = 24$ is the cardinality of H. The number of constraints is:

$$14 \cdot |H| + 2 \cdot |A| + \sum_{a \in A} (\theta_a \cdot (\beta_a - \alpha_a) + |H| + \alpha_a + \theta_a - \beta_a - 1)$$

where $|A|$ is the cardinality of A, i.e., the number of schedulable loads. For example, considering a number of schedulable loads equal to 5, an average working time θ equal to 3 hours and an average user preference interval $(\beta - \alpha)$ equal to 10, the number of resulting constraints is 576.

In order to find a correct solution for the *unified prosumer problem*, it is necessary to impose the z_a^h variables as binary, so the whole optimization problem can be classified as a mixed integer linear programming problem[1]. This kind of problem can be solved with the Branch and Bound technique, in particular we exploited the implementation provided by the CPLEX Library[2]. CPLEX allows to stop the execution of the algorithm after a predetermined number of iterations or execution time, and provide a sub-optimal solution if the optimal one has not been found. However, in our scenario, given the limited size of the problem, we did not set any limit to the execution because the algorithm is able to achieve the optimal solution in a few seconds.

IV. EXPERIMENTAL RESULTS

In this Section, we evaluate the effectiveness of the presented unified model. In particular, we compare the annual costs/revenues supported by a prosumer, in two cases: *CASE I* considers the separated electrical and thermal management, also referred to as non-unified model in the following; *CASE II* considers the unified electrical/thermal management. The optimization model is solved by using the Java language and the CPLEX library. The optimal solution of the model is computed in a time interval between 1 and 3 seconds on an ASUS Intel I7 computer equipped with 16GB RAM and Windows 10. The results are evaluated on three prosumers, equipped with identical electrical equipments and different thermal generation plants. Each prosumer owns a dwelling with a size of about 100 m^2, which hosts four people and is located in the municipality of Rende, Italy. Table I reports the user preferences, about the schedulable loads, considered for the experiments. In Table II, the production plants and the storage systems of each prosumer are reported with their main technical characteristics. The dwellings need a daily amount of electrical energy of about 10 kWh, for each day of the year. The daily thermal energy requirement depends on the season: it is about 20.0 kWh in spring and autumn (assuming a set point temperature T equal to 23°C), about 8.7 kWh in summer ($T = 25°C$) and about 42.0 kWh in winter ($T = 20°C$).

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[1] In a mixed integer linear programming problem, some variables are constrained to be integer (or binary) and some others are not

[2] https://www.ibm.com/us-en/marketplace/ibm-ilog-cplex

TABLE I
USER SETTINGS FOR THE SCHEDULABLE LOADS

Schedulable loads	**Rated Power (W)**	θ	α	β
Washing machine	1500	2	11	18
Tumble dryer	1200	1	16	22
Dish washer	1900	1	10	17
Electric vehicle	200	3	9	19

TABLE II
PLANTS OF THE PROSUMERS

	Prosumer 1	**Prosumer 2**	**Prosumer 3**
Photovoltaic plant with a peak power of	$6kW$	$6kW$	$6kW$
Connection to the grid with a rated power of	$6kW$	$6kW$	$6kW$
Electric storage system with a capacity of	$6kWh$	$6kWh$	$6kWh$
Solar panel plant with an area of	$4m^2$	$4m^2$	$4m^2$
Heat pump with a rated power of	$10kWt$	-	-
Natural gas boiler with a rated power of	-	$25kWt$	-
Combined generator with rated powers of	-	-	$2kWe$ and $6kWt$
Thermal storage system with a volume of	$250l$	$250l$	$250l$

Table III reports the daily revenues and costs experienced by the three prosumers in the different seasons, when the electrical and thermal management are separated (*CASE I*). More specifically, the table reports the cost of the electrical energy imported from the grid, the revenue obtained by selling the electrical energy to the grid, the cost of the natural gas, the cost of the biomasses and finally the overall daily revenue[3]. The electrical equipment usage is scheduled by solving the prosumer problem in which only the electrical part is considered. The scheduling for the thermal part is obtained by considering that the thermal load of a prosumer is fulfilled by the solar panel and by the controllable thermal plant owned by the prosumer. Table IV shows the results obtained when the unified electrical/thermal model is adopted (*CASE II*). In this case, Prosumer 1 registers an annual revenue of 295.50 €, Prosumer 2 presents an annual cost of 6.30 €, and Prosumer 3 has an annual cost of 12.54 €.

TABLE III
CASE I: DAILY REVENUES AND COSTS OF A PROSUMER

		Electric energy cost (€)	**Electric energy revenue (€)**	**Natural gas cost (€)**	**Biomasses cost (€)**	**Daily revenue/cost (€)**
Prosumer 1	*Spring/Autumn*	1.000	1.441	-	-	0.441
	Summer	0.288	1.474	-	-	1.186
	Winter	1.335	0.419	-	-	-0.916
Prosumer 2	*Spring/Autumn*	0.287	1.411	0.793	-	0.331
	Summer	0.287	1.474	0.011	-	1.176
	Winter	0.351	0.723	2.746	-	-2.374
Prosumer 3	*Spring/Autumn*	0.287	1.577	-	0.987	0.303
	Summer	0.287	1.476	-	0.014	1.175
	Winter	0.351	1.281	-	3.417	-2.487

TABLE IV
CASE II: DAILY REVENUES AND COSTS OF A PROSUMER

		Electric energy cost (€)	**Electric energy revenue (€)**	**Natural gas cost (€)**	**Biomasses cost (€)**	**Daily revenue/cost (€)**
Prosumer 1	*Spring/Autumn*	0.309	1.411	-	-	1.102
	Summer	0.287	1.474	-	-	1.187
	Winter	0.864	0.723	-	-	-0.141
Prosumer 2	*Spring/Autumn*	0.287	1.411	0.566	-	0.558
	Summer	0.287	1.474	0	-	1.187
	Winter	0.351	0.723	2.745	-	-2.373
Prosumer 3	*Spring/Autumn*	0.287	1.542	-	0.704	0.551
	Summer	0.287	1.474	-	0	1.187
	Winter	0.263	1.247	-	3.417	-2.433

TABLE V
COMPARISON BETWEEN CASE I AND CASE II

		Daily improvement (%)	**Case I Annual Revenue (€)**	**Case II Annual Revenue (€)**	**Annual Improvement (%)**
Prosumer 1	*Spring/Autumn*	149,9	103,68	295,50	182,00
	Summer	0,0			
	Winter	-84,6			
Prosumer 2	*Spring/Autumn*	68,6	-48,24	-6,30	-86,90
	Summer	0,9			
	Winter	0,0			
Prosumer 3	*Spring/Autumn*	81,8	-63,54	-12,54	-79,6
	Summer	1,0			
	Winter	-2,2			

In Table V, the results of the prosumer management in *CASE I* and *CASE II* are summarized. In particular, the table shows the additional revenues or cost savings obtained with the unified model with respect to those associated with the non-unified model. We can notice that the unified model offers a significant advantage for Prosumer 1, especially in spring/autumn and in winter. Indeed, this prosumer can obtain a reduction of the electrical energy cost and an increase of the electrical energy, as can be seen by comparing the data reported in Table III and in Table IV. In general Table V shows that, when using the unified model, all the prosumers experience an economic improvement, especially in spring/autumn when the limited thermal load is mainly supplied by the generation from nPRS plants. Indeed, the unified model allows to face an excess of production from nPRS plants with respect to the energy demand and to store this excess into the storage systems. The unified model schedules the charge/discharge of storage systems depending on the energy load and generation, and determines when it is convenient to inject/absorbe electrical energy into/from the grid.

It is worth to underline that the HP maximizes the interaction between thermal and electric management because it acts as an electrical load and thermal generator at the same time. More specifically, the HP allows to use cheap electric energy, e.g., energy produced by the PV plant or low-price energy imported from the grid, to produce the thermal energy.

The PV production profile is obtained from the forecasting services implemented in the Cloud Service Provider, as described in [21]. The electricity price is taken from the Italian day-ahead market[4]. Figures 3 and 4 show, respectively, the electrical equipment scheduling and the thermal equipment scheduling of Prosumer 1 in a typical spring/autumn day. Figure 3 also shows the hourly energy costs, i.e., the selling price and the purchasing price, and the PV profile retrieved from the Cloud service provider. In the figure the "distribution grid" indicates the exchange profile between the nanogrid of Prosumer 1 and the grid: negative values represent an absorption (purchase) of energy, while positive values represent an injection (sale) of energy.

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[3] A positive value corresponds to a revenue while a negative value corresponds to a cost

[4] www.mercatoelettrico.org

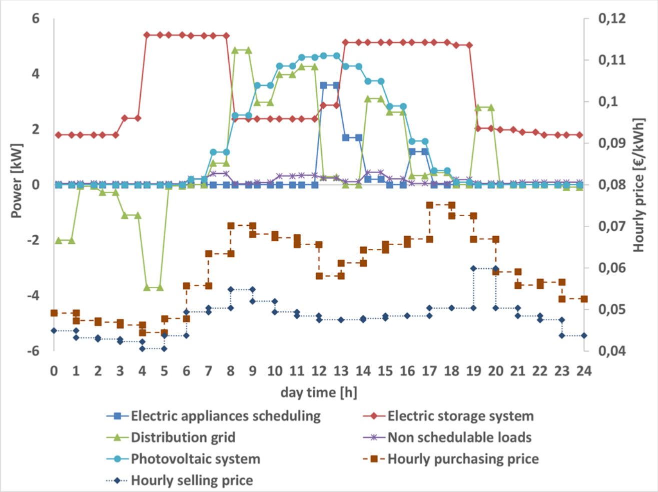

Fig. 3. Electrical scheduling for Prosumer 1 in a spring/autumn day

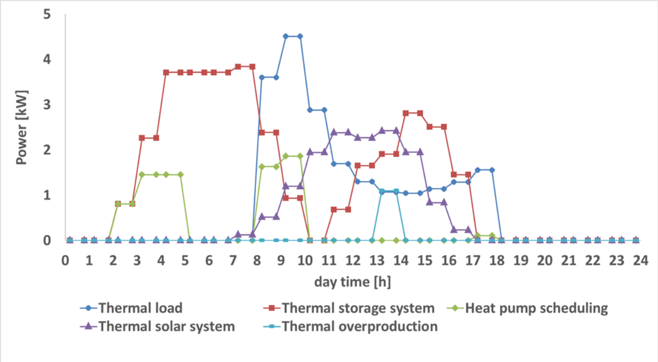

Fig. 4. Thermal scheduling for Prosumer 1 in a spring/autumn day

Several interesting considerations can be taken from Figure 3, as described in the following. At 4:00, an absorption of energy from the grid completely recharges the electric storage system, because the purchasing price is minimum. At 8:00, the energy produced from the PV plant and the stored energy are partially injected into the grid due to the high selling price. The electric storage system remains at the minimum SOC level until 12:00, because the PV production is sufficient to supply the schedulable and non-schedulable loads and the energy surplus is injected into the grid. The electric storage system is recharged from the PV plant from 12:00 to 14:00, because the value of the selling price is at a local minimum, so the management model decides to store energy and sell it later, in a more convenient time, specifically starting from 19:00.

Figure 4 shows that, starting at 8:00, the HP and the thermal storage system supply the required thermal load. More specifically, from 8:00 to 9:00, the HP exploits the energy provided by the PV plant and the electric storage system. At 10:00, the thermal storage system is completely discharged and the thermal load is supplied from the thermal solar system. From 11:00 to 14:00, the production from the thermal solar system that is not required by the thermal load is stored into the thermal storage system.

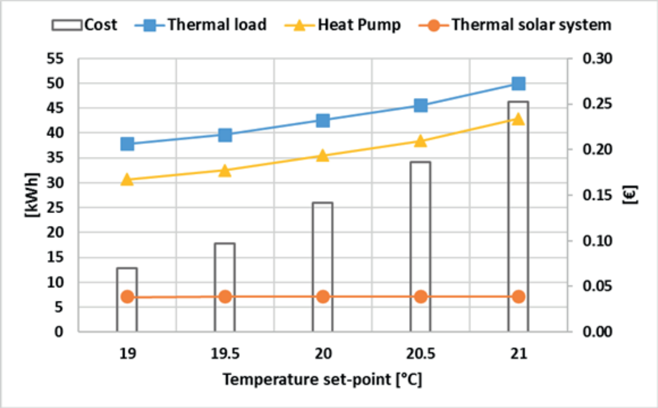

Fig. 5. Daily cost and thermal balance for a typical winter day, for Prosumer 1, when varying the temperature set point.

As a conclusion, it is interesting to assess the results of the unified prosumer problem when varying the user preferences. As an example, we executed the prosumer problem with different values of the temperature set point desired by the Prosumer 1 in a typical winter day. We recall that the results shown in Tables III, IV and V were obtained with a set point temperature equal to $20°C$. We assumed that the user can decide to change the temperature in a range between $19°C$ and $21°C$. In Figure 5 we show the main thermal values and the costs related to the entire day, for the different values of the temperature set point. Specifically, we can see that the thermal load increases with the desired temperature. Since the energy guaranteed by the solar system is constant, the additional energy is supplied by the heat pump. The figure also shows that the daily cost increases with the desired temperature. In particular, we can see that the daily cost increases from about €0.14 to about €0.25 when the prosumer increases the temperature set point from $20°C$ to $21°C$, while it decreases to about €0.07 when the temperature set point is decreased to $19°C$.

V. CONCLUSIONS

This paper has presented a novel approach for the optimal management of prosumers, which can autonomously set their own requirements and constraints, and are coordinated and linked to the distribution grid by an aggregator. The paper has introduced and described in detail the "unified prosumer problem", modeled as a mixed integer linear optimization problem and solved with a Branch and Bound algorithm. The solution of the problem consists in the definition of the usage schedule of electrical and thermal appliances and of renewable-based generators, allowing each prosumer to maximize the revenues and minimize the costs while respecting the local constraints. The efficiency and novelty of the approach mainly rely on two features: the first concerns the concurrent management of electrical and thermal energy, which leads to a significant cost saving or revenue increase when compared to the approaches where the two aspects are managed separately; the second pertains to the definition of the energy district architecture, which combines the computational power of a

Cloud service provider, in charge of computing aggregated information and supplying it to the prosumers, with the limited but distributed power of end-user smart energy boxes. The advantages have been assessed through a testbed performed under an academic/industrial Italian project.

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