On Possible Connections between Ant Algorithms and Random Matrix Theory

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This paper reports on a conjecture concerning the statistical behavior of Self-Chord [1], a self-organizing P2P system in which the resource keys are dynamically sorted with an ant algorithm. In Self-Chord (http://self-chord.icar.cnr.it), peers are organized in a logical ring, as in Chord, and a hash function is used to assign an index to every peer, and an access key to every resource. Contrary to Chord though, the values of resource keys are decoupled from those of peer indexes, and are dynamically sorted by ant-inspired agents through statistical *pick* and *drop* operations. This allows Self-Chord to keep the Chord capacity for serving discovery requests in logarithmic time, but leads to many further advantages, among which the possibility of assigning a semantic meaning to keys, a better load balancing among peers, and the efficient execution of range queries.

Figure 1 reports a sample snapshot of a Self-Chord network, in which peer indexes and resource keys are defined over 6 and 3 bits, respectively. At the interior of the ring, the figure specifies the indexes of the peers, whereas at the exterior it reports, for every peer, some of the keys stored by the peer, and the peer centroid. The latter is defined as the real value that minimizes the average distance between itself and the keys stored by the peer. Both the values of centroids and peer indexes are sorted in clockwise direction, but they are not related to one another. Indeed, different approaches are used to sort them: the peer indexes are sorted by Chord-like management operations, whereas the keys are dynamically sorted by the operations of the Self-Chord agents.

Interestingly, it emerged that the statistical distribution of the peer centroids is very similar to the distribution of the eigenvalues of random matrices taken from the GUE, Gaussian Unitary Ensemble. These matrices are used to model a wide class of complex dynamical systems, especially in the domain of nuclear physics [3]. The GUE matrices are also the subject of the Montgomery-Odlyzko law, which states that the distribution of the spacings between the non-trivial zeros of the Riemann Zeta function is statistically identical to the spacings of GUE eigenvalues. Figure 2 reports a comparison between the theoretical distribution of GUE spacings and the distribution of centroid spacings in Self-Chord networks with a number of peers N_p equal to 2000, 5000 and 10000. The two distributions are very similar, and the similarity increases with the size of the network. This observation is also supported by several qualitative considerations on the similarity between the behavior of Self-Chord centroids and that of the energy levels of physical systems modeled by GUE matrices [2].

The Self-Chord algorithm is very similar to many ant-inspired sorting algorithms (see the book on Swarm Intelligence authored by Bonabeau et al.).

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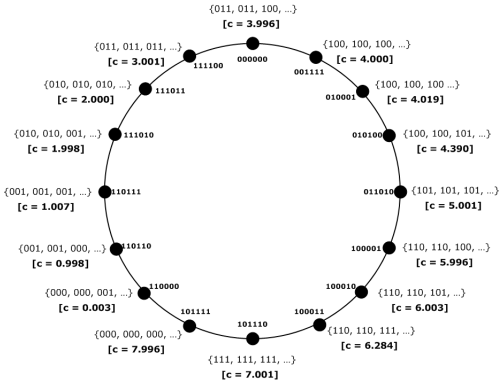

Fig. 1. Sample Self-Chord network. For each peer, its index, a number of stored keys and the centroid are reported.

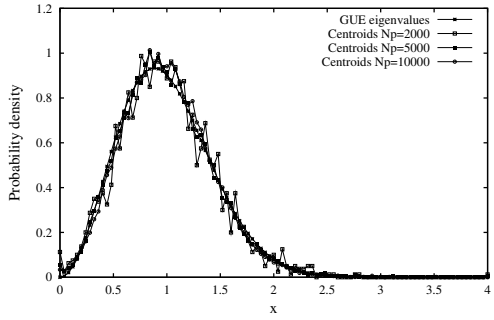

Fig. 2. Comparison between the distribution of spacings between consecutive centroids in Self-Chord and the theoretical distribution of spacings between GUE eigenvalues

Therefore, it is plausible that this similarity, if confirmed, could apply to ant algorithms in general, not only to Self-Chord. This would suggest the hypothesis that the mathematical nature of ant algorithms is inherently connected to random matrix theory and, more widely, to number theory. It is the opinion of the author that this fascinating conjecture is worth being analyzed more deeply and with more rigorous tests. More information can be found in [2].

References

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